

# TRANSIENT MULTIPHASE MULTICOMPONENT FLOW IN POROUS MEDIA†

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**Abstract**—A one-dimensional transient two-phase, two-component flow in a porous medium is investigated analytically. The flow is described by the use of a continuity equation for each component, a momentum equation for each phase, the first law of thermodynamics, state equations, and relations for the transport properties. Local thermodynamic equilibrium is assumed. The effects of condensation and evaporation are included. The analysis shows that the flow resulting from a step change in the state at the surface of a uniform semi-infinite porous medium may be described by a single variable, which is a simple function of position and time. Under other conditions, position and time must be separately specified to determine the state of the mixture. Results of calculations relevant to the flow of cavity gas following an underground nuclear explosion are given.

## NOMENCLATURE

$e$ ,	internal energy per unit mass;
$h$ ,	enthalpy per unit mass;
$k$ ,	permeability;
$k_r$ ,	relative permeability;
$L$ ,	length of porous bed;
$p$ ,	pressure;
$p^*$ ,	inlet pressure;
$S$ ,	saturation;
$t$ ,	time;
$u$ ,	apparent speed;
$x$ ,	position coordinate in direction of flow;
$\varepsilon$ ,	porosity;
$\mu$ ,	viscosity;
$\mu^*$ ,	viscosity of air at ambient temperature;
$\rho$ ,	density.

## Dimensionless quantities

$P$ ,	dimensionless pressure, $p/p^*$ ;
$X$ ,	dimensionless position $x/L$ ;
$\theta$ ,	similarity variable, $X/2\sqrt{\tau}$ ;
$\tau$ ,	dimensionless time, $kp^*t/\varepsilon\mu^*L^2$ .

## Subscripts

$a$ ,	air;
$l$ ,	liquid;
$m$ ,	air-water vapor mixture;
$s$ ,	solid;
$v$ ,	water vapor;
$0$ ,	air at ambient conditions.

## INTRODUCTION

THE CAVITY formed by an underground nuclear explosion ceases growing about a tenth of a second after the explosion. The vapor in the cavity consists predominately of a mixture of rock gas and steam at a temperature of a few thousand degrees Kelvin and a pressure near the overburden pressure. The cavity gas ordinarily remains at temperatures and pressures of this magnitude until collapse of the cavity, which usually occurs after several minutes. During this period, should the cavity gas have access to either the porous medium surrounding the cavity or the stemming column, the gas will flow into the porous material. The purpose of this paper is to present the results of analysis of such a transient multiphase multicomponent flow.

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### FORMULATION OF THE PROBLEM

In recent years, the analysis of transient multiphase flow in porous media has received considerable attention. The subject has been of particular interest to the petroleum industry because of its importance to the secondary recovery of petroleum. As a result, most of the available literature on such flows, e.g. [1-12], deals with conditions that may be found in petroleum reservoirs. Multiphase flow resulting from an underground nuclear explosion, however, passes through considerably larger ranges of temperature and pressure. Consequently, assumptions in these studies usually prohibit their application to the problem of interest here. For example, the fluids are ordinarily considered incompressible [1-4] or barotropic [5-10]. Although such restrictive assumptions are not made in [11, 12], the formulation of the first law of thermodynamics is in error in both cases.

The flow will be assumed to be a two-phase two-component flow. When applying the analysis to flow of the cavity gas, the two components are taken to be air and water. Air is initially present in the porous medium. The gas flowing from the cavity is predominantly steam. Because the temperature of the gas entering the porous medium is considerably higher than the ambient temperature of the medium, condensation will occur and a two-phase flow results.

The equations governing this flow may be formulated in a straight forward manner. The equations will be written for a one-dimensional flow, the extension to more dimensions being obvious.

Conservation of mass is expressed by a continuity equation for each component. For air, this equation is

$$\frac{\partial}{\partial x}(\rho_a u_m) + \varepsilon \frac{\partial}{\partial t}(\rho_a S_m) = 0. \quad (1)$$

Here, air is assumed to be present only in the vapor phase. Any air dissolved in the liquid phase is negligible.  $x$  is the distance in the direction of flow from the surface of the porous

medium.  $t$  is the time.  $\rho_a$  is the mass density of the air.  $u_m$  is the apparent velocity, i.e. volume flow rate/unit area normal to flow, of the mixture of air and water vapor. The two phases will ordinarily have different apparent velocities.  $S_m$  is the saturation of the vapor phase. The saturation of a phase is the void volume fraction occupied by that phase.

The corresponding equation for water is

$$\frac{\partial}{\partial x}(\rho_l u_l + \rho_v u_m) + \varepsilon \frac{\partial}{\partial t}(\rho_l S_l + \rho_v S_m) = 0. \quad (2)$$

The additional terms result from the presence of water in both phases. The subscript  $l$  refers to the liquid and the subscript  $v$  refers to water vapor. The air and water vapor have the same apparent velocity, diffusion being neglected.

The flow results from the pressure gradient in the medium. Flow due to gravity is not considered. For sufficiently small Reynolds numbers, so that inertial effects are negligible, the apparent velocity of a phase will be given by Darcy's law with the permeability reduced by the presence of the other phase.

$$u_l = - \frac{k_{r,l} k}{\mu_l} \frac{\partial p}{\partial x} \quad (3)$$

$$u_m = - \frac{k_{r,m} k}{\mu_m} \frac{\partial p}{\partial x}. \quad (4)$$

A characteristic Reynolds number for this flow is calculated later.

The reduction in permeability is expressed by use of the relative permeability.  $k_{r,l}$  and  $k_{r,m}$  are the relative permeabilities of the liquid and the vapor mixture, respectively. The relative permeability of a phase is the ratio of the effective permeability for the phase to the absolute permeability,  $k$ , of the medium. Although not strictly functions of saturation only, the relative permeabilities depend most strongly on saturation and are ordinarily taken to depend only on saturation. The functional form of the dependence is different for the two phases because of the wetting of the liquid phase.  $\mu$  and  $p$  are viscosity and pressure respectively.

For an isothermal flow, these equations, together with state and transport property relations and the initial and boundary conditions, would be sufficient to describe the flow. A noncondensable gas flow, under the conditions considered here, could be treated as isothermal with very little loss of accuracy. The temperature of the incoming gas from the cavity is very much larger than the initial temperature of the porous medium. As the gas flows, the temperature at any location in the medium will rise. However, before this change in temperature can become appreciable at a position, the mass of gas passing that position must be of a magnitude near to the mass of the solid lying between that position and the inlet. Consequently, for noncondensable gas flow, the time required to produce significant changes in the temperature is extremely long compared to the time in which changes in pressure become significant. This gas flow is, for all practical purposes, isothermal, since the gas rapidly comes to the bed temperature. Morrison [13] has analyzed transient ideal gas flow under these conditions.

In the flow of interest here, however, condensation will occur. The temperature in the porous medium will rise more rapidly as a result. Moreover, the partial pressure of the water vapor cannot exceed the saturation pressure at the local temperature. Until the temperature at a position begins to change significantly, the steam will exert little pressure there. We conclude that, in this case, the local temperature and fluid pressure will begin to change rapidly at the same time and that this change will result from local condensation of the steam.

In order to describe this flow, it is necessary to apply the first law of thermodynamics. Because the response time for local heat transfer from the fluid to the solid grains is several orders of magnitude smaller than the times of interest, this response time may be neglected. Thus, the fluid and solid temperatures are assumed equal. This condition of local thermodynamic equilibrium simplifies the formulation of first law,

avoids ambiguities in the designation of local temperatures, and makes a model for heat transfer between fluid and solid superfluous. The assumption of temperature equality is common in analysis of flows through beds of small particles, e.g. *vid.* [11-15].

For an elemental slab with surfaces normal to the direction of flow, the first law is written

$$\frac{\partial}{\partial x} (\rho_l u_l h_l + \rho_v u_m h_v + \rho_a u_m h_a) + \varepsilon \frac{\partial}{\partial t} (\rho_l S_l e_l + \rho_v S_m e_v + \rho_a S_m e_a) + (1 - \varepsilon) \rho_s \frac{\partial e_s}{\partial t} = 0. \quad (5)$$

$e$  and  $h$  are the internal energy and enthalpy per unit mass, respectively. The subscript  $s$  refers to the solid. Changes in kinetic energy are neglected in agreement with the low Reynolds number restriction implicit in the use of Darcy's law. Conduction of heat through the bed is neglected in comparison to the heat transfer by the motion of the fluid. This, too, is a frequently used assumption in the analysis of flow through porous media [16, 17]. The thermal conductivity of unconsolidated sand [18] is sufficiently small that this mechanism of heat transfer becomes important only after long times as the rate of fluid flow decreases.

Defining a few dimensionless quantities facilitates the numerical solution of these equations and permits easy comparison of the results with results of corresponding ideal gas flows. Accordingly, we define a dimensionless position,

$$X = x/L \quad (6)$$

a dimensionless pressure,

$$P = p/p^* \quad (7)$$

and a dimensionless time

$$\tau = kp^*t/\varepsilon\mu^*L^2. \quad (8)$$

$L$  is the length of the bed.  $p^*$  is the highest pressure at the inlet (simply the inlet pressure

for the step change boundary condition employed here) and  $\mu^*$  is the viscosity of the air in the medium at ambient conditions. If the permeability and porosity of the medium are not uniform and constant, then reference values of these properties should also be used. We will not consider such variations here. The difference between the definition of  $\tau$  used here and that used in [13] is small when the inlet pressure is large. Results of the two analyses may be readily compared.

Elimination of the apparent velocities from the continuity equations and the first law is accomplished by substitution of the two Darcy's law expressions into these equations. With these substitutions, we obtain relations that contain only thermodynamic and transport properties as dependent variables. Making these substitutions and introducing the dimensionless variables, we obtain for air continuity,

$$\frac{\partial}{\partial X} \left( \frac{\rho_a k_{rm} \mu^*}{\mu_m} \frac{\partial P}{\partial X} \right) = \frac{\partial}{\partial \tau} (\rho_a S_m) \quad (9)$$

for water continuity

$$\frac{\partial}{\partial X} \left[ \left( \frac{\rho_l k_{rl} \mu^*}{\mu_l} + \frac{\rho_v k_{rm} \mu^*}{\mu_m} \right) \frac{\partial P}{\partial X} \right] = \frac{\partial}{\partial \tau} (\rho_l S_l + \rho_v S_m) \quad (10)$$

and for the first law

$$\frac{\partial}{\partial X} \left\{ \left[ \frac{\rho_l h_l k_{rl} \mu^*}{\mu_l} + (\rho_v h_v + \rho_a h_a) \frac{k_{rm} \mu^*}{\mu_m} \right] \frac{\partial P}{\partial X} \right\} = \frac{\partial}{\partial \tau} \left( \frac{1-\varepsilon}{\varepsilon} \rho_s e_s + \rho_l S_l e_l + \rho_v S_m e_v + \rho_a S_m e_a \right). \quad (11)$$

Equations(9)–(11) govern the one-dimensional transient two-phase two-component flow through a finite porous bed with uniform permeability and porosity. Initial conditions, boundary conditions, state and transport properties remain to be specified.

Boundary conditions at the inlet to the porous medium are given by the state of the gas within

the cavity. In an underground nuclear experiment, the cavity pressure attains a value near the overburden pressure after cessation of cavity growth. The temperature in the cavity is of the order of a few thousand degrees Kelvin. Cavity conditions then ordinarily change slowly until the collapse of the cavity. Cavity collapse may occur several minutes after the detonation. Olsen [19] describes the cavity pressure and temperature history resulting from an underground nuclear experiment in alluvium.

It is convenient to consider idealized boundary conditions corresponding to step changes of cavity temperature and pressure to constant values. Results of flow calculations, using step change boundary conditions, can be presented in a particularly simple and useful form. The advantage extends until such time as significant property changes begin to occur near the other end of the porous medium.

If we restrict our attention to those times when the effects of flow from the cavity cause only insignificant changes at the far boundary, the far boundary may be removed and the porous medium considered to be semi-infinite. The time during which the effect of a distant boundary is negligible may be easily estimated from the resulting calculations. A numerical example is presented later.

The advantage of considering flow in a semi-infinite medium resulting from a step change in the inlet conditions is due to the existence of a similar solution. Defining a dimensionless variable

$$\theta \equiv \frac{X}{2\sqrt{\tau}} = \frac{x}{2} \sqrt{\left( \frac{\varepsilon \mu^*}{k_p t} \right)} \quad (12)$$

we find that the partial differential equations (9)–(11) describing the flow may be written as ordinary differential equations in the independent variable,  $\theta$ . The only restriction is that all dependent variables must be functions solely of the local thermodynamic state. No property may be taken, *a priori*, to have an explicit dependence on either position or time. In terms

of the similarity variable, equations (9)–(11) then become

$$\frac{d}{d\theta} \left( \frac{\rho_a k_{rm} \mu^*}{\mu_m} \frac{dP}{d\theta} \right) + 2\theta \frac{d}{d\theta} (\rho_a S_m) = 0, \quad (13)$$

$$\begin{aligned} \frac{d}{d\theta} \left[ \left( \frac{\rho_l k_{rl} \mu^*}{\mu_l} + \frac{\rho_v k_{rm} \mu^*}{\mu_m} \right) \frac{dP}{d\theta} \right] \\ + 2\theta \frac{d}{d\theta} (\rho_l S_l + \rho_v S_m) = 0 \end{aligned} \quad (14)$$

and

$$\begin{aligned} \frac{d}{d\theta} \left\{ \left[ \frac{\rho_l h_l k_{rl} \mu^*}{\mu_l} + (\rho_v h_v + \rho_a h_a) \frac{k_{rm} \mu^*}{\mu_m} \right] \frac{dP}{d\theta} \right\} \\ + 2\theta \frac{d}{d\theta} \left( \frac{1-\varepsilon}{\varepsilon} \rho_s e_s + \rho_l S_l e_l \right. \\ \left. + \rho_v S_m e_v + \rho_a S_m e_a \right) = 0 \end{aligned} \quad (15)$$

respectively.

Boundary conditions at the cavity–medium interface correspond to conditions at  $\theta$  equal to zero. Only step change boundary conditions may be expressed in this manner. A similar solution does not exist for less idealized cavity conditions. In those cases, the local thermodynamic state in the medium is not a function of the single variable,  $\theta$ , but depends on position and time separately.

Initial conditions in the medium correspond to conditions as  $\theta$  goes to infinity. Only uniform initial conditions in a semi-infinite medium may be so expressed. Note that the length of the bed, present in the definitions, (6) and (8), of dimensionless position and time, does not appear in the definition (12) of  $\theta$ . Because the changes of state become quite small for sufficiently large finite values of  $\theta$ , the results may be applied to flow in finite beds for significant periods of time.

The solution of the governing set of equations, (9)–(11) or (13)–(15), is best accomplished by the use of numerical techniques. One may readily demonstrate that a non-trivial closed form analytical solution in terms of well-tabulated functions does not exist. If we consider only the relatively simple special case of isothermal ideal gas, then the flow is governed by equation (13)

alone since no water is present and the temperature is constant. In this limiting case, equation (13) can be considerably simplified. With only a single phase present, the relative permeability and saturation are each unity. When the temperature is constant and no water vapor is present,  $\mu_m$  is everywhere equal to  $\mu^*$ . Finally, the air density is proportional to the pressure and equation (13) reduces to

$$\frac{d}{d\theta} \left( P \frac{dP}{d\theta} \right) + 2\theta \frac{dP}{d\theta} = 0. \quad (16)$$

Using the integral transformation of von Mises [20], this equation may be transformed to Blasius equation. Because we obtain Blasius equation in this limiting case, we verify that closed form solutions may not be obtained for the flows governed by equations (13)–(15).

#### NUMERICAL SOLUTION

The existence of a similar solution does not imply that the ordinary differential equations in the similarity variable are easier to solve numerically than the partial differential equations which have position and time as independent variables. Because any advantage in numerically solving the ordinary differential equations is not apparent, and because a program to solve the partial differential equations can be used under less restrictive conditions, a program was written to solve the partial differential equations (9)–(11). Results of calculations using this program may be presented in the similar form simply by using the value of  $\theta$  corresponding to the values of  $X$  and  $\tau$ . The program is explicit and self-starting. The finite difference expressions employed have truncation errors of the order of the step size. Use of higher order expressions results in the appearance of a Gibbs phenomenon because of shocklike features of the flow.

The solution of equations (9)–(11) is accomplished by first calculating the finite difference approximations of the derivatives on the left hand sides of these equations. The equations are then numerically integrated in

time for one time step. This yields values of the three quantities in parentheses on the right hand sides of equations (9)–(11) at the next discrete value of time. In accordance with Gibbs' phase rule, three such quantities are necessary and sufficient information to determine the state of the mixture, including the amounts of water present as liquid and as vapor.

If the water present at a location is a superheated vapor, then the expressions for the three calculated quantities are significantly simplified by setting the water saturation and vapor mixture saturation equal to zero and one, respectively. Determination of the state is then straightforward. If the calculated values of the three quantities are inconsistent with a superheated water state, the system contains a mixture of saturated liquid and saturated vapor. The state is then determined iteratively by using successive linear interpolation to find the temperature. Saturation properties, the relations for air and solid internal energies, and the quantity determined from equation (9) are substituted into the expressions for the quantities that were found by integration of equations (10) and (11). Two estimates of the water saturation result. The iteration proceeds toward the temperature at which the water saturation estimates agree, yielding the state of the system.

Subsidiary equations to calculate state and transport properties must also be used. Air is considered to be an ideal gas with a variable specific heat. The viscosity of air is considered to be a function only of temperature and is calculated using an empirical correlation given by Hilsenrath and Touloukian [21].

Because of limitations on computation time, the model for water properties is simpler, but less accurate, than other formulations, e.g. [22], generally available. The pressure of saturated vapor is calculated using the relation of Keenan *et al.* [23]. The vapor is assumed to obey Clausius equation of state. The constant in the Clausius equation of state is chosen so that calculated values of the density of saturated vapor agree well with tabulated values. The

agreement, of course, is poor in the neighborhood of the critical point. A relation given by Keenan and Keyes [24] is used to determine the density of saturated liquid. Variable specific heats are used for liquid and vapor. The viscosities of liquid water and steam are taken to be functions only of temperature. The viscosity of the liquid is very accurately represented in the temperature range of interest by taking this viscosity to be proportional to the inverse Celsius temperature. The empirical correlation of Hilsenrath and Touloukian [21] for steam viscosity is used. Note that the value 33.15 appearing in the correlation in [21] is a misprint and should read 3315.0.

The porous medium is considered incompressible with constant specific heat, permeability and porosity. The viscosity of the air–water vapor mixture is calculated from the viscosities, molecular weights and mole fractions of the two species. The relation for the mixture viscosity is due to Wilke [25].

The relative permeabilities of the liquid and vapor phases are calculated using relations proposed by Wyllie and Gardner [26] for liquid and gas flow in porous materials. The relations of Wyllie and Gardner have been simplified by assuming constant capillary pressure. The relative permeabilities are taken to be functions only of saturation. When, because of evaporation, the water saturation becomes less than the "irreducible" water saturation, the relative permeabilities of the liquid and vapor phases are taken to be zero and one, respectively. The resulting relative permeability curves lie within the broad band of experimental results given by Wyckoff and Botset [27] for flow of gas–liquid mixtures through unconsolidated sand.

#### RESULTS OF SAMPLE CALCULATION

As an example of the type of flow that would result from an underground nuclear detonation, we may consider idealized cavity conditions based on the measurements of Olsen [19]. The cavity pressure is taken to be a suddenly applied constant pressure of 45 bars. The cavity tem-

perature is taken to be 1710°K. These values are in good agreement with values deduced for the Rainier event from examination of core samples [28]. Water in the cavity is superheated under these conditions. Since any rock vapor will rapidly condense, the gas in the cavity may be considered to be predominately steam. In the example, the assumed cavity gas composition is 90% steam and 10% air by mass.

The initial temperature of the porous medium is chosen to be 310°K. The porosity of the medium is 0.35 and the medium is assumed to be initially dry.

The results of numerical calculations of flow under these conditions are shown in Figs. 1-4.

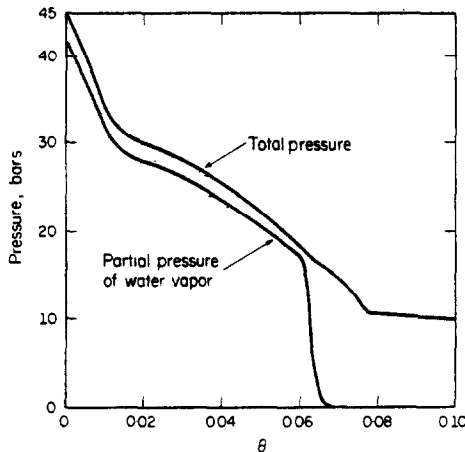


FIG. 1. Pressure distribution in infinite medium.

The abscissae of Figs. 1-3 have identical scales. These figures show the nature of the flow in the region where effects of condensation, evaporation and multiphase flow are important. Figure 4 shows the pressure distribution over a larger range and, for comparison, shows results of calculations for isothermal ideal gas flows. Further discussion of Fig. 4 is deferred until consideration of Figs. 1-3 is completed.

The nature of the flow in the region where the presence of water is important is perhaps best perceived by simultaneously considering Figs. 1-3, which show pressure and partial pressure, water saturation, and temperature distributions, respectively. Since the properties are shown as

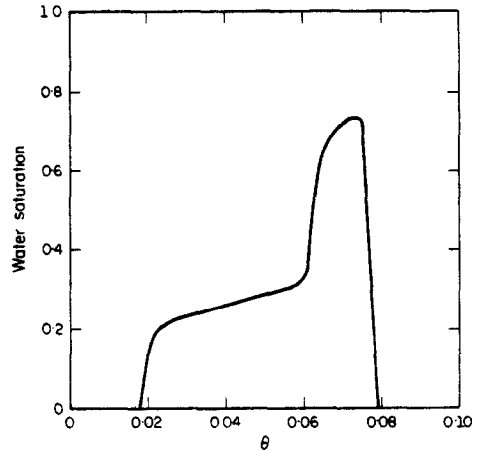


FIG. 2. Water saturation distribution in infinite medium.

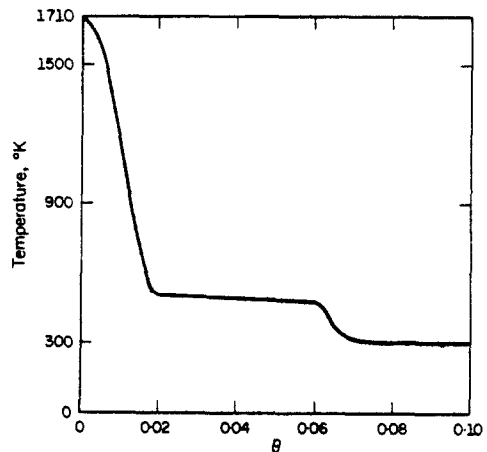


FIG. 3. Temperature distribution in infinite medium.

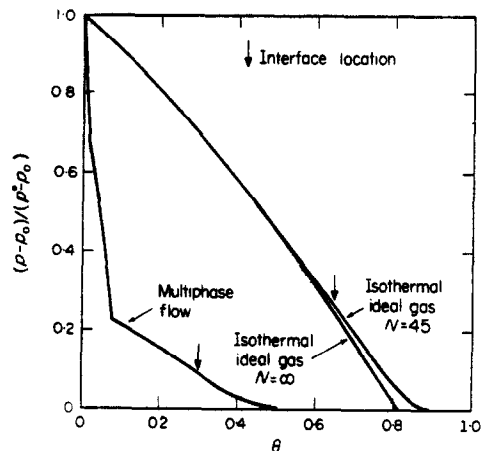


FIG. 4. Pressure distribution in infinite medium.

functions of the similarity variable,  $\theta$ , these curves represent the spatial distributions of these properties at any time. Given a time, the scale of the abscissae may be converted to the corresponding linear scale showing distance from the inlet. Values of position are determined by reference to the definition of  $\theta$ , equation (12). Conversely, these figures may be used to show the temporal variation of state at any given location. This transformation is nonlinear as equation (12) shows.

The flow may be characterized as having several distinct regimes. Nearest the inlet to the porous bed, the water exists as a superheated vapor. In the example shown, this regime extends from the inlet, where  $\theta$  is equal to zero, to a value of  $\theta$  roughly equal to 0.02. Because no liquid is present, the water saturation, shown in Fig. 2, is zero here. The decrease in total pressure and partial pressure, as seen in Fig. 1, is accompanied by the decrease in temperature shown in Fig. 3.

The flow then enters a second regime where a mixture of liquid and water vapor is present. The temperature is relatively constant through most of this regime, changing slowly in the region bounded by  $\theta$  equal to 0.02 and 0.06. Changes in the partial pressure of the water vapor here correspond to changes of the saturation pressure with temperature. The change in total pressure is largely determined by the change of the water vapor pressure. The water saturation changes relatively slowly through most of this region. The positive slope of the water saturation curve indicates that the fractional pore volume occupied by liquid at a bed position in this region decreases with time due to evaporation and the flow of liquid towards regions of lower pressure.

From  $\theta$  approximately equal to 0.06 to  $\theta$  nearly equal to 0.08, the temperature of the bed again decreases rapidly with distance from the inlet. At  $\theta$  near 0.08, the temperature is essentially the initial bed temperature. The increase in temperature with time at a bed location in this region is due to the motion of liquid

and condensation of the vapor. The vapor condenses rapidly as it enters this cooler region. The appearance of a sharp condensation front or water saturation front, as in Fig. 2, is commonly encountered in the steam or water flooding of petroleum reservoirs, e.g. *vid.* [29]. After this sudden appearance of liquid at a location producing a high water saturation, the water saturation then decreases with increasing time (or decreasing  $\theta$ ). The rate of decrease is more rapid here than in the region where  $\theta$  ranges from 0.02 to 0.06 because of the large relative permeability of the liquid in this range of water saturation. The rapid change in the vapor pressure, shown in Fig. 1, results from the temperature change. The vapor pressure does not vanish near the front of this region but has values corresponding to the saturation pressures at temperatures near ambient.

Beyond the water saturation front,  $\theta$  greater than about 0.08, no liquid exists. Water, in small amounts, exists beyond this point as superheated vapor. The flow is almost entirely a flow of air. The heat capacity of the air is insufficient for air flow to cause significant changes in the local temperature. Consequently, the flow is, in essence, an isothermal ideal gas flow of air at the initial temperature of the porous medium. Referring now to Fig. 4, the pressure distribution of the multiphase flow described here is shown in dimensionless form. The saturation front is located at the near discontinuity in the pressure gradient. The pressure drop in the regime of isothermal gas flow is very gradual compared to that in the two-phase regime preceding it.

For ready comparison of the multiphase flow calculations with calculations based on an isothermal ideal gas model, the dimensionless pressure distributions for two isothermal ideal gas flows are also shown in Fig. 4. These flows correspond to ratios of inlet pressure to ambient pressure,  $N$ , of infinity and 45. Transient isothermal gas flow under these conditions was analyzed in [13], where a slightly different definition of  $\theta$  is used. Because of the absence of condensation, these gas flows propagate more



rapidly than the multiphase flow. Because the isothermal ideal gas calculations are much faster than the multiphase calculations, the ideal gas model provides a convenient means of rapidly determining a bound for the flow into the porous medium.

The "interface location" whose position is noted on Fig. 4 is the position of the interface between gas from the cavity and gas originally in the bed when a piston-like displacement is posited. The location is determined by a mass balance. The analysis providing a simple means of finding the interface location is given in the following section. The interface for the infinite pressure ratio isothermal ideal gas flow is located at  $\theta$  equal to 0.81 where the dimensionless pressure goes abruptly to zero.

As an example of the application of these analyses, let us predict the actual times and distances associated with certain occurrences. The infinite pressure ratio ideal gas analysis predicts the effects of the flow to extend up to  $\theta$  equal to 0.81. The multiphase flow considered here has the water saturation front near  $\theta$  equal to 0.08. Referring to the definition (12) of  $\theta$ , choose  $\epsilon$  to be 0.35,  $p^*$  to be 45 bars, and  $\mu^*$  to be  $189 \mu P$  as in the multiphase flow computations. Further select a permeability,  $k$ , of one darcy (1 darcy =  $9.8 \times 10^{-9} \text{ cm}^2$ ), typical of stemming materials used in these tests. Substitution into equation (12) then gives the distance of the water saturation front from the cavity as 1 m, 3.2 m, and 25 m at times of 1 min, 10 min and 10 h, respectively. At these same times,  $\theta$  equal to 0.81 corresponds to distances of roughly 10 m, 32 m and 250 m.

While no single Reynolds number describes this flow, it is worthwhile to determine the Reynolds number in at least one instance so as to indicate the range of Reynolds number. The Reynolds number based on particle diameter can be readily calculated for the gas flow near the saturation front as an example. Taking a particle diameter of 1 mm, characteristic of stemming material, this Reynolds number is found to be approximately 0.5 when the front

is 1 m from the cavity. This corresponds to a time of 1 min. For longer times, the Reynolds number at this saturation front will be smaller. For shorter times, the Reynolds number will of course be much larger. High Reynolds number effects are confined to small times and distances compared to those of interest.

#### EXTENT OF PENETRATION

The extent of cavity gas penetration into the porous medium is a matter of considerable interest. Fluid from the cavity clearly extends beyond the saturation front since all of the water was initially in the cavity. The extent to which air from the cavity has traveled is less obvious. The air in the medium consists of air initially in the medium as well as air that entered from the cavity. The flow will result in some mixing of these gases because of dead spaces, non-uniform velocity distribution across open areas, the distribution of path lengths, etc. Danckwerts [30] has examined incompressible flows with a distribution of residence times in considerable detail.

If mixing may be neglected, then the gas from the cavity will entirely displace the gas originally in the bed. This model corresponds to a piston-like displacement and is commonly used in the analysis of flows through packed beds. Air from the cavity would be separated from air initially in the porous medium by an interface which moves through the bed. The location of this interface may be determined by a simple extension of the method employed by the author in [13]. Because of the presence of some mixing, the interface location corresponds to a lower bound for the distance from the inlet of the leading gas originating in the cavity.

In the similarity analysis of [13], the interface location is readily determined from the pressure distribution because an isothermal flow is barotropic. The local gas density and the accumulated mass of gas in any region are obtained directly from the pressure and its spatial integral in that region. Although the multiphase flow of interest here is not barotropic

throughout, it becomes barotropic in the region that is considered in finding the interface. This region is that portion of the bed that lies ahead of the water saturation front. As we have seen, the interface location must lie in this region and the flow there is, with considerable accuracy, an isothermal flow of air.

As in [13], the interface location corresponds to that bed position where the mass of air originally lying closer to the inlet has been added to the gas lying further from the inlet

$$\int_x^{\infty} (\rho - \rho_0) \varepsilon dx = \rho_0 \varepsilon x. \quad (17)$$

The subscript 0 denotes air at ambient conditions.

Considering air as an ideal gas, and recognizing that the temperature beyond the interface is constant and equal to the initial bed temperature, this relation may be written

$$\int_{\theta}^{\infty} (P - P_0) d\theta = P_0 \theta. \quad (18)$$

The definitions of dimensionless variables are different from those used in [13].

Although equations (13)–(15) are needed for a complete description of the multiphase flow, equation (13) reduces to equation (16) and equations (14) and (15) are identically satisfied in the region beyond the interface. This simplification results immediately from the constant temperature and absence of water in that region.

Equation (16) is used to evaluate the integral in equation (18) using substitutions analogous to those of [13]. The interface location is again found to be the position where

$$\frac{dP}{d\theta} + 2\theta = 0 \quad (19)$$

or

$$\frac{d^2P}{d\theta^2} = 0. \quad (20)$$

The interface is located at the inflection point of the pressure distribution curve as in Fig. 4.

## CONCLUSIONS

Multiphase multicomponent flow issuing from a cavity following an underground nuclear experiment has been investigated analytically. A program was developed and results obtained for the one-dimensional transient flow of air, steam, and water through a uniform unfractured porous medium. A numerical example, using idealized but meaningful conditions, was presented.

These results were compared with those obtained from an isothermal ideal gas model and reveal the extent to which such an isothermal ideal gas model is conservative in containment calculations.

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#### ÉCOULEMENT TRANSITOIRE À PLUSIEURS PHASES ET PLUSIEURS COMPOSANTS DANS UN MILIEU POREUX

**Résumé**—On étudie analytiquement un écoulement transitoire unidirectionnel biphasique et à deux composants dans un milieu poreux. L'écoulement est décrit en utilisant une équation de continuité pour chaque composant, une équation de quantité de mouvement pour chaque phase, la première loi de la Thermodynamique, des équations d'états et des relations pour les propriétés de transport. On suppose l'équilibre thermodynamique local et on inclut les effets de la condensation et de l'évaporation. L'analyse montre que l'écoulement résultant d'un changement échelon de l'état à la surface d'un milieu poreux uniforme semi-infini peut être décrit par une variable unique qui est une fonction simple de la position et du temps. Dans d'autres conditions, position et temps peuvent être spécifiés séparément pour déterminer l'état du mélange. On donne des résultats de calculs applicables à l'écoulement des gaz qui succède à une explosion nucléaire souterraine.

#### MEHRPHASIGE MEHRKOMONENTEN-ÜBERGANGSTRÖMUNG IN PORÖSEN MEDIEN

**Zusammenfassung**—Eine eindimensionale, zweiphasige und aus zwei Komponenten zusammengesetzte instationäre Strömung in einem porösen Medium wird analytisch untersucht. Die Strömung wird durch die Kontinuitätsgleichung für jede einzelne Komponente, eine Momenten-Gleichung für jede Phase, den ersten Hauptsatz der Thermodynamik, Zustandsgleichungen und Beziehungen zwischen den Transportgrößen beschrieben. Örtliches thermodynamisches Gleichgewicht wird angenommen. Die Effekte von Kondensation und Verdampfung sind mit eingeschlossen.

Die Untersuchung zeigt, dass die aus einer schrittweisen Zustandsänderung an der Oberfläche eines homogenen, halbunendlichen porösen Mediums resultierende Strömung durch eine einzige Variable beschrieben werden kann, die eine einfache Funktion von Ort und Zeit ist.

Unter anderen Bedingungen müssen Ort und Zeit getrennt bestimmt werden, um den Zustand des Gemisches zu bestimmen. Die auf die Strömung von Gas in Kavernen als Folge einer unterirdischen Atomexplosion anwendbaren Rechenergebnisse werden angegeben.

#### НЕСТАЦИОНАРНОЕ МНОГОФАЗНОЕ МНОГОКОМПОНЕНТНОЕ ТЕЧЕНИЕ В ПОРИСТЫХ СРЕДАХ

**Аннотация**—Аналитически исследуется одномерное нестационарное двухфазное двухкомпонентное течение в пористой среде. Течение описывается с помощью уравнения неразрывности для каждого компонента, уравнения количества движения для каждой фазы, первого закона термодинамики, уравнений состояния и соотношений для переносных свойств. Предполагается локальное термодинамическое равновесие. Анализ показывает, что течение, вызванное скачкообразным изменением состояния на поверхности однородной полубесконечной пористой среды, может быть описано одной переменной, являющейся простой функцией координат и времени. В других условиях для описания состояния смеси координату и время следует определять отдельно. Приводятся результаты расчета течения газа в полости, являющегося следствием подземного ядерного взрыва.